INSTANTANEOUS ORBITS FOR BINARY ASTEROIDS

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Abstract

We developed the theoretical framework for the calculation of orbits for a binary system, consisting of a spherical and a non-spherical objects. A computer program was written for the special cases where the non spherical body is a prolate spheroid. A particular trajectory is presented.

1. INTRODUCTION

Data from occultations (Binzel and Van Flandern 1979) and lightcurves (Wijenghe and Tedesco 1979) of asteroids, as well as the observations of mission Galileo, suggest satellites around asteroids are not uncommon. These discoveries and the exploration of Solar system bodies by unmanned missions have made the study of orbiting satellite of irregular shaped bodies an interesting topic. In this paper we will discussed the problem of computation of orbits for binary systems consisting of a spherically symmetric body m_1 and a body, m_2, of arbitrary symmetry.

The attraction of a solid homogeneous ellipsoid upon a test particle is obtained from Multon (1914). We then applied the model to the case where m_2 represents a homogenous prolate asteroid, which could be considered as a starting point in the case where m_2 represents a homogeneous prolate asteroid, which could be considered as a starting point in the investigation of more realistic situations.

2. THE TWO BODY PROBLEM

From Newton's second law it follows that the acceleration of the center of mass of a body m_1 relative to the second body, m_2, center of mass is given by

$$\frac{d^2r}{dt} = \frac{F}{m} \qquad (1)$$

where $r(x,y,z)$ is the vector position of the center of mass of m_1 relative to an origin at the center of mass of body m_2, $m = m_1m_2/(m_1+m_2)$ is the reduced mass, an F is the gravitational force on m_1 produced by the interaction with m_2 . In general, F would be a function of (x,y,z) and the orientation in space of the two bodies. However, in our situation m_1 is spherically symmetric, and therefore only the rotational degrees of freedom of m_2 counts. This orientation can be conveniently described by the well known set of three Eulerian angles

(f, q, y) given by Goldstein (1980), where the coordinate system (x', y', z') axes are fix in m_2, while (x,y,z) axes are fix in space, see figure 1. Thus we have, writing equation 1. Thus we have in component form,

$$\frac{d^2 x_i}{dt^2} = F_{xi}(x, y, z, f, q, y) \qquad (2)$$
$$x_1 = x, \quad x_2 = y, \quad x_3 = z$$

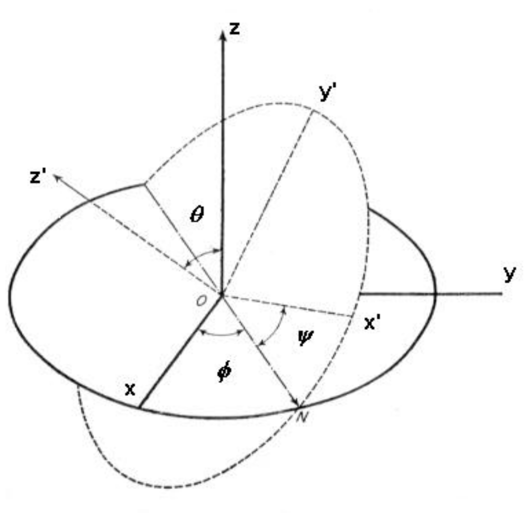

Figure 1: The rotations defining the Eulerian angles

Since the above three differential equations are in six unknown we need three additional equations to be able to integrate the equations of motion and find the orbits. We will furnish these additional equations using the conservation law for total angular momentum which is valid for isolated systems. For a two body problem where one of them is a sphere one can write without loss of generality,

$$L = mr \times \frac{dr}{dt} + I \bullet w = L_0 \ (3)$$

where the first term represents the orbital angular momentum of the system, and $l = I \bullet w$ is the intrinsic angular momentum of m_2. I and w are the tensor of inertia and angular velocity vector in the x, y, z coordinate system respectively, and we have ignored the intrinsic angular momentum of m_1, since for spherically symmetric bodies is a constant of the motion that can be put equal to zero.

It is possible to choose the coordinate axes x', y', z' in the body such that the tensor of inertia is reduced to a diagonal form. These directions are called “the principal axes of inertia”, and the corresponding values of the diagonal components of the tensor are called “the principal moments of inertia”: $I_{x',x'}$, $I_{y',y'}$, $I_{z',z'}$, and thus we have

$$I' = \begin{bmatrix} I_{x'x'} & 0 & 0 \\ 0 & I_{y'y'} & 0 \\ 0 & 0 & I_{z'z'} \end{bmatrix} \qquad (4)$$

The intrinsic angular momentum in the principal axes of inertia, , $l' = I' \bullet w'$ is related to l by the transformation

$$l' = Al \qquad (5)$$
$$l = A^{-1} l' = A^{-1} I' \bullet w' \qquad (6)$$

where A is the Euler matrix, which transforms vector components in the x,y,z coordinate system to vector components in the x',y',z' system and is given by Goldstein (1980):

$$A = \begin{pmatrix} \cos y \cos f - \cos q \sin f \sin y & \cos y \sin f + \cos q \cos f \sin y & \sin y \sin q \\ -\sin y \cos f - \cos q \sin f \cos y & -\sin y \sin f + \cos q \cos f \cos y & \cos y \sin q \\ \sin q \sin f & -\sin q \cos f & \cos q \end{pmatrix} \qquad (7)$$

Furthermore the angular velocity w' along the axes x',y',z' can be expressed in terms of the Eulerian angles and their time derivative as follows

$$w' = \begin{pmatrix} w_{x'} \\ w_{y'} \\ w_{z'} \end{pmatrix} = B \begin{pmatrix} df/dt \\ dq/dt \\ dy/dt \end{pmatrix} \qquad (8)$$

where, according to Goldstein (1980),

$$B = \begin{pmatrix} \sin q \cos y & \cos y & 0 \\ \sin q \cos y & -\sin y & 0 \\ \cos q & 0 & 1 \end{pmatrix} \qquad (9)$$

Hence from Eq. 6 we obtain

$$l = A^{-1} I' B \begin{pmatrix} df/dt \\ dq/dt \\ dy/dt \end{pmatrix} \qquad (10)$$

which implies that

$$\begin{pmatrix} df/dt \\ dq/dt \\ dy/dt \end{pmatrix} = B^{-1I'^{-1}} Al = B^{-1} I'^{-1} A \begin{pmatrix} l_x \\ l_y \\ l_z \end{pmatrix} \qquad (11)$$

On the other hand from equation 3 we have that

$$l = L_0 - mr \times \frac{dr}{dt} \qquad (12)$$

and then

$$\begin{aligned} l_x &= L_{0x} - m\left[y\frac{dz}{dt} - z\frac{dy}{dt} \right] \\ l_y &= L_{0y} - m\left[z\frac{dx}{dt} - x\frac{dz}{dt} \right] \\ l_z &= L_{0z} - m\left[x\frac{dy}{dt} - y\frac{dx}{dt} \right] \end{aligned} \qquad (13)$$

Consequently equations (11) give three additional differential equations in the unknown x, y, z, f, q, y and their time derivatives and therefore together with the three equations in (2) can be integrated to provide the orbit of m_1 and the orientation of m_2 as a function of time.

3. PROCEDURE AND RESULTS

We have written a program to perform the above integration numerically and applied it to the case where m_2 is an homogenous prolate spheroid. The expression for the force for an homogeneous prolate is calculated using the transformation.

$$\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = A^{-1} \begin{pmatrix} F_{x'} \\ F_{y'} \\ F_{z'} \end{pmatrix} \qquad (14)$$

Where $F_{x'}$, $F_{y'}$, $F_{z'}$ are the components of the force along the principal axes of inertia, which are given by Multon (1914).

An example of a trajectory is given in figure 2, where we show the evolution of a small object orbit around a prolate body during a time of days. The bodies are homogeneous with a mass density of 2300 kg/m^3. The shape of m_2, is a prolate, its major axis, *Cz*, is equal to 52 km and the eccentricity is 0.75. The initial rotational period is two hours, which is a typical value for small asteroids. The smaller body is a sphere with a radio of 0.25 km orbiting at a distance between about 1.3 and 12 *Cz*. The origin of coordinates is the center of mass of m_2, The magnitude of the initial velocity and position of m_1 are 10 m/s and 6.7 Cz respectively.

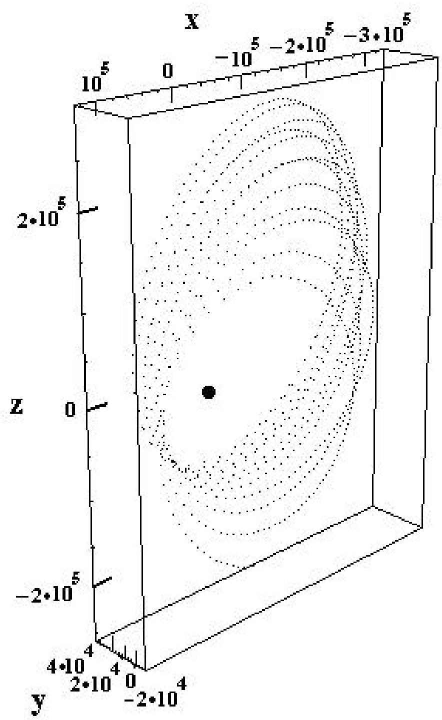

Figure 2: Orbit evolution of a small satellite around a prolate body. The scale is in meters.

The trajectory of figure 2 is represented by 1000 points, each one is separated by a time interval of 1000 second. The integration time interval is of one second, then the orbit shown had evolved during 11.57 days. This $\overrightarrow{OA}$instantaneous" orbit show fast changes in eccentricity and also in the position of the orbital plane. Although the rotation effect is not showing here, for this model the main body initial rotation is affected by the orbital interaction. This example could be view as a model for the known asteroid Ida and its little moon Dactyl, which is a system with a lot yet to be learn.

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