

Wavelet Application to Structural Dynamics

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Abstract

This paper presents the research of the wavelet application for the damage identification in structures. Simply supported beam with a transverse on-edge nonpropagating open crack was chosen as a case of study. The results show that some wavelets were able to detect exact crack location and that regularity is an important characteristic in selecting the appropriate wavelet.

1. Introduction

Everywhere around us are signals that need to be analyzed. Seismic tremors, engine vibrations, human speech, music and many other types of signals have to be efficiently encoded, compressed, cleaned up, reconstructed, simplified, modeled, distinguished, or located.

Signal analyses already have at their disposals an impressive arsenal of tools. Fourier analysis, short time Fourier transform, and wavelet theory are three different levels of signal processing.

- **Fourier Analysis**

The most well known of these techniques is the Fourier analysis, which breaks down a signal into constituent sinusoids of different frequencies. Another way to think of Fourier analysis is as a mathematical technique for transforming our view of the signal from a time-based one to a frequency-based one.

Fourier analysis has a serious drawback. In transforming to the frequency domain, time information is lost. When looking at a Fourier transform of a signal, it is impossible to tell when a particular event took place.

- **Short-Time Fourier Analysis**

In an effort to correct this deficiency, Dennis Gabor (1946) adapted the Fourier transform to analyze only a small section of the signal at a time - a technique called windowing the signal. Gabor's adaptation,

called the Short-Time Fourier Transform (STFT), maps a signal into a two-dimensional function of time and frequency. The STFT represents a sort of compromise between the time-and frequency-based views of a signal. It provides some information about both when and at what frequencies a signal event occurs. However, one can only obtain this information with limited precision, and that precision is determined by the size of the window.

- **Wavelet Analysis**

Wavelet analysis represents the next logical step: a windowing technique with variable-size regions. Wavelet analysis allows the use of long time intervals where one wants more precise low frequency information, and shorter regions where one wants high frequency information.

One major advantage afforded by wavelets is the ability to perform local analysis, that is, to analyze a localized area of a larger signal. Wavelet analysis is capable of revealing aspects of data that other signal analysis techniques miss, aspects like trends, breakdown points, and discontinuities. Furthermore, because it affords a different view of data than those presented by traditional techniques, wavelet analysis can often compress or de-noise a signal without appreciable degradation. Indeed, in their brief history within the signal-processing field, wavelets have already proven themselves to be an indispensable addition to the analyst's collection of tools.

2. History of Wavelets

From an historical point of view, wavelet analysis is a new method, through its mathematical underpinnings date back to the work of Joseph Fourier in the nineteenth century. Fourier laid the foundations with his theories of frequency analysis, which provide to be enormously important and influential. The attention of researches gradually turned from frequency-based analysis to scale-based analysis when it started to become clear that an approach measuring average fluctuations at different scales might prove less sensitive to noise.

The existence of wavelet-like functions has been known since the early part of century. The first recorded mention of the term "wavelet" was in 1909, in a thesis by Alfred Haar.

The concept of wavelets in its present theoretical form was first proposed by Jean Morlet and the team at the Marseille Theoretical Physics Center working under Alex Grossmann in France.

The methods of wavelet analysis have been developed mainly by Y. Meyer and his colleagues, who have ensured the method's dissemination. The most popular algorithm dates back to the work of Stephane Mallat in 1988. Since then, research on wavelets has become international. Such research is particularly active in the United States, where it is spearheaded by the work of scientists such as Ingrid Daubechies, Ronald Coifman, and Victor Wickerhauser.

3. Objectives

The aim of the proposed research is to apply the wavelet analysis to the identification of damage in structures.

Cracks in structural elements cause local variations in stiffness that can considerably affect the dynamic behavior of the structure. Changes in natural frequencies, modes of vibrations and dynamic stability occur because of the existence of such cracks. However, the effect of a small crack may not be evident from changes in the eigenfrequencies of the structure or from the first several eigenfunctions. Thus, detection of cracks in structures in a more effective and accurate manner is still an important area of current research.

The main concept is to break down the dynamic signal of a structural response in a series of space domain to its different series of local basis functions so as to detect some of the special characteristics of the structure.

A high degree of resolution can not be obtained in both the time and frequency domains simultaneously using the short-time Fourier transform.

This problem has led to the study of the application of wavelets in signal processing. Wavelet analysis is a fundamentally different approach. Basically, the theory is based on the idea that any signal can be broken down into a series of local basis functions called "wavelets ". Any particular local feature of a signal can be analyzed based on the scale and transformation characteristics of wavelets.

4. Procedure

A beam with a transverse on-edge nonpropagating open crack was chosen as a case of study (Fig. 1). The stiffness of the beam is affected by the presence of cracks. The crack introduces a discontinuity in the deflection and slope as well as in the time response signal, which makes it possible to detect it by wavelets.

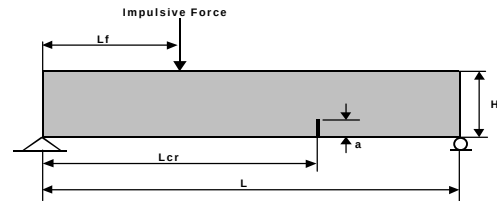


Fig. 1 Beam with nonpropagating open crack

4.1 Wavelet Characteristics

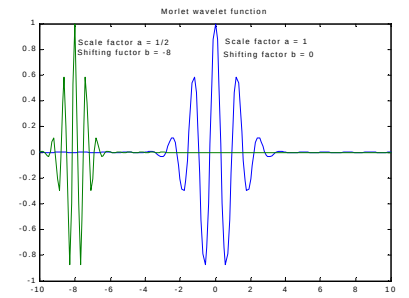
There are many different wavelets that can be created. Wavelets are composed of a family of basis functions that are capable of describing signals in a localized time and frequency (or scale) format. The sets of basis functions have "compact support" meaning all their energy is localized to a finite space in time.

Wavelet analysis starts with a basic wavelet function called the "mother wavelet", "generating wavelet" or "prototype wavelet". The prototype function is then shifted and scaled (or dilated) to generate a family of orthonormal basis functions (Fig. 2). All the functions defined in the space spanned by the wavelet family can then be modeled by a linear combination of shifted and scaled prototype wavelets.

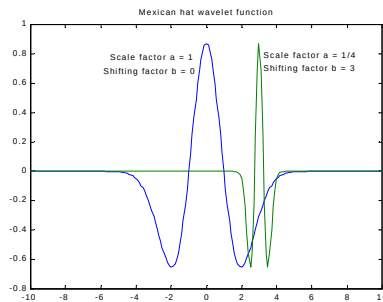
4.2 Wavelet Decomposition

The response signal "displacement of the nodes for fixed time" (Fig. 3) was analyzed using the Discrete Wavelet Transform, whose main idea is described below.

By using wavelet analysis, a signal is decomposed into a hierarchical set of approximations and details, i.e., a family of hierarchically organized decompositions. The selection of suitable level for the hierarchy will depend on the signal and experience.



a) **Morlet Wavelet Function**



b) **Mexican Hat Wavelet**

Fig. 2 Scaling and Shifting of Mother Wavelets

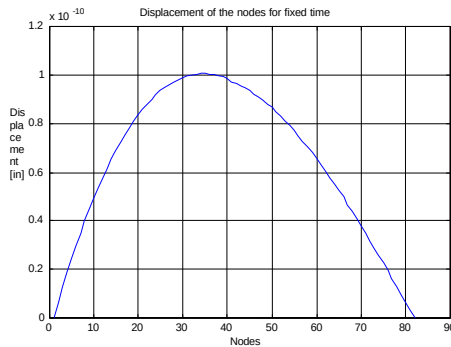


Fig. 3 Time Response History

At each level j , we build the j -level approximations A_j , or approximations at level j , and a derivation signal called the j -level detail, D_j , or detail at level j . The original signal can be considered as the approximation at level 0, denoted by A_0 . The words "approximation" and "detail" are justified by the fact that A_1 is an approximation of A_0 taking into account the "low frequencies" of A_0 , whereas the detail D_1 corresponds to the "high frequency" corrections.

5. Results and Conclusions

After performing the Discrete Wavelet Transform of the time response history of the cracked beam (Fig. 3), it was found that some wavelets were able to detect the crack location. The numerical studies performed showed that not all the wavelets could detect the signal discontinuity and that regularity is an important criterion in selecting the proper wavelet.

The sampled signal "displacement of the nodes for fixed time" (Fig. 3) was analyzed using two wavelets: Haar, that is insufficiently regular, and bior6.8, that is sufficiently regular. By analyzing the signal with the Haar wavelet (Fig. 4a) one can notice that the singularity is not being detected. However, with bior6.8 (Fig. 4b), the discontinuity is clearly detected. One can conclude that the selected wavelet must be sufficiently regular, which implies a longer filter impulse response in order to detect the singularity.

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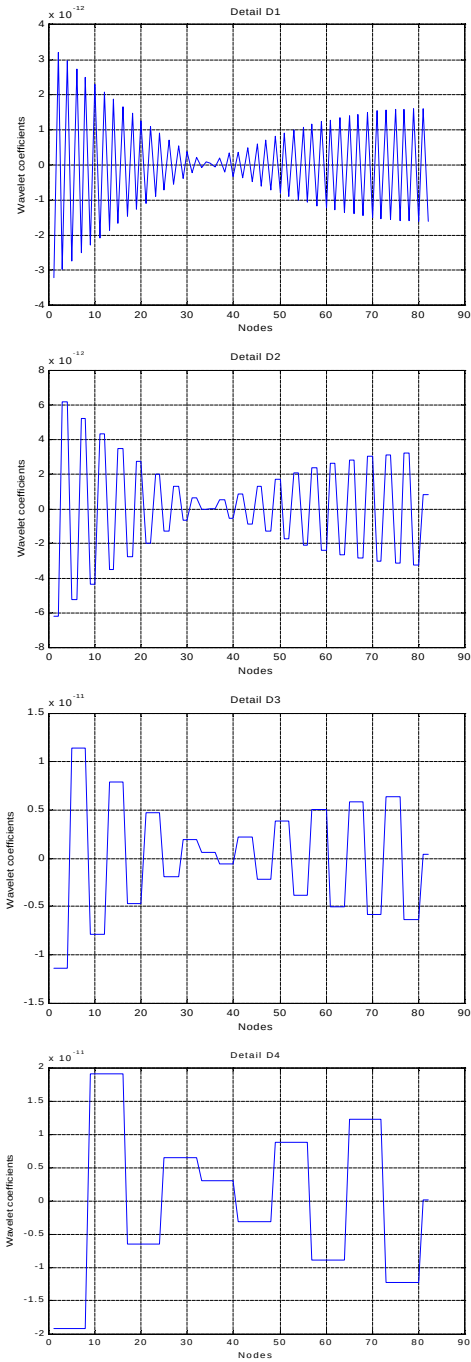


Fig. 4 a) Haar 4-levels Decomposition

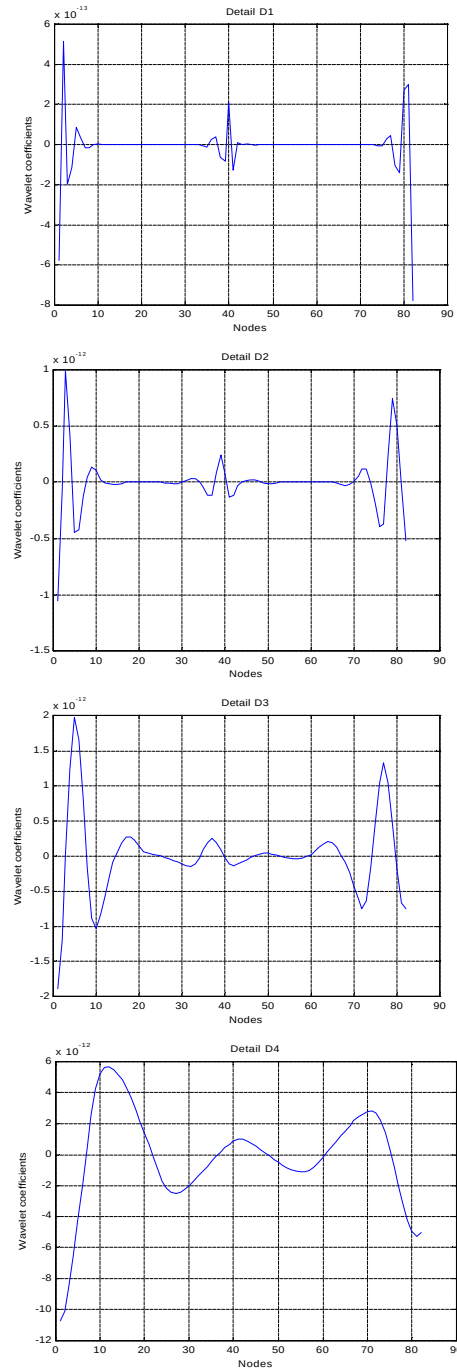


Fig. 4 b) Bior6.8 4-levels Decomposition

Fig. 4 Four-Levels Wavelet Decomposition