A Formulation for the Rate of Change of Seismic Structural Responses With Respect to Characteristics of Added Dampers

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Abstract

This study considers the use of dampers in buildings to improve their structural behavior during a seismic event. For this purpose, a new formulation is presented herein. This formulation estimates the seismic response of a particular design quantity and its first derivative with respect to the damping coefficient of an added damper. Since both, the response itself and its first derivative are quite useful for sensitivity analysis, it is expected that the availability of the proposed formulation will become a valuable design tool. Also, it is worth mentioning that the proposed formulation requires the seismic forces to be defined in terms of ground response spectra that are widely used by the earthquake engineering community.

1. INTRODUCTION

When a building with added dampers is subjected to earthquake forces, part of the induced seismic energy is dissipated at the dampers and less energy is left to damage the structure. Therefore, the proper addition of dampers to a new or existing structure may substantially improve its general seismic performance. However, its behavior cannot longer be described by a classical analysis. A non-classical (non-proportional damping) analysis is now required.

Non-proportional structures had been analyzed by many researches. Different approaches to calculate the dynamic responses of those systems have been proposed. The techniques using response spectrum methods consider three different formulations: the mode displacement (MD) formulation [Singh (1980), Igusa, Der Kiureghian and Sackman (1984), Villaverde (1988)], the mode acceleration (MA) formulation [Singh and McCown(1986)] and the Modified Mode Displacement (MMD) formulation [Maldonado and Singh (1992)]. The MD approach was used by Bizier (1990) to estimate the seismic response of a design quantity and its first derivative. This was done for a shear-beam model when the input motion was defined in terms of power spectral density functions (PSDF). In this work, we also estimate the response and its first derivative using similar expressions to those used by Bizier. However, we now employ the MMD approach with ground response spectra as input. Also, we plan to implement it to model 3D frame structures subjected to the ground response spectra required for Puerto Rico by the Uniform Building Code (1997).

There are different ways to attach dampers to a structure. The following pictures show the most commonly used locations:

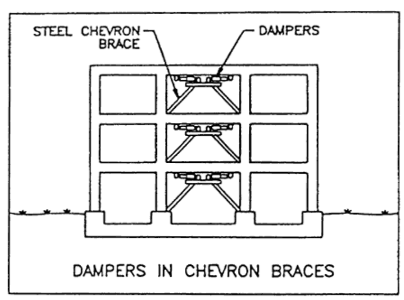

Figure 1. Example of dampers in structures (Taylor Devices)

The addition of this kind of dampers to the structure will simply and efficiently raise structural damping to 20% - 50% of critical, versus 1%-5% for a typical undamped design. Diagonal brace dampers are available in output levels of 2,000 pounds to 2,000,000 pounds force, with strokes of up to 6 inches. Base isolation dampers are available in output levels of 100,000 pounds to 2,000,000 pounds force, with strokes of up to 120 inches.

An example of this damper is shown in figure 2. The one shown is a damper of 14 inches in diameter, 12 feet in length, with an available stroke of plus or minus 24 inches. Each damper dissipates some 3,000 horsepower under maximum earthquake conditions.

Figure 2. Example of a damper from Taylor Devices, Inc.

2. FORMULATION

2.1. Dynamic Response of the Structure.

The equations of motion for a structure subjected to seismic forces is:

$$[M]\left\{\ddot{X}(t)\right\}+[C]\left\{\dot{X}(t)\right\}+[K]\left\{X(t)\right\}=-[M]\{I\}\ddot{x}_g(t)$$

Where: $[M]$ is the consistent mass matrix, $[K]$ is the stiffness matrix and $[C]$ is the proportional damping matrix. $\{I\}$ is the influence vector, $\ddot{x}_g(t)$ is the ground acceleration and $\left\{\dot{X}(t)\right\}$ is the vector of structural displacements relative to the ground.

The addition of dampers results in a non-classical damping matrix, i.e. in a non-proportional damping matrix. This matrix $[C]$ models the classical structural damping and the added dampers as well.

To implement a proper modal decomposition, it is now necessary to employ the state vector approach Therefore; the second order system is transformed into the following first order system:

$$[A]\left\{\dot{Y}(t)\right\}+[B]\left\{Y(t)\right\}=-[D]\begin{Bmatrix}\{0\}\\ \{I\}\end{Bmatrix}\ddot{x}_g(t)$$

Where $\{Y(t)\}$ is the state vector defined as:

$$\{Y(t)\}=\left\{\left\{\dot{X}(t)\right\}\ \left\{X(t)\right\}\right\}^T$$

and:

$$[A]=\begin{bmatrix}[0] & [M]\\ [M] & [C]\end{bmatrix},\qquad [B]=\begin{bmatrix}-[M] & [0]\\ [0] & [K]\end{bmatrix},\qquad [D]=\begin{bmatrix}[0] & [0]\\ [0] & [M]\end{bmatrix}$$

To decouple this equation we obtain and use the complex eigenproperties of the following associate eigenvalue problem:

$$l_j[A]\{f\}_j=[B]\{f\}_j\ ;\quad j=1,\ldots\ldots.,2n$$

The solution of these expressions provides $2n$ eigenvalues l_j and their corresponding $2n$ eigenvectors $\{f\}_j$. Because of the characteristics of the system, these properties are given in n pairs of complex and conjugate quantities. However, the MMD approach only requires a few retained eigenproperties, i.e. only r pairs ($r<n$) need to be used to obtain a good estimate of the maximum response values and their rates.

After invoking the orthogonality properties, the original first order equation can be decoupled for the principal coordinates $z_j(t)$ as:

$$\dot{z}_j(t)+l_j z_j(t)=-g_j\ddot{x}_g(t)\qquad j=1,\ldots\ldots,2n$$

where g_j is the jth participation factor.

For quiescent initial conditions, the last equation can be solved for all $z_j(t)$ to get:

$$z_j(t)=-g_j\int_0^t\ddot{x}_g(t)e^{-l_j(t-t)}dt\qquad j=1,\ldots.,2n$$

This solution, can in turn be used to calculate the relative displacement vector as follows:

$$\{X(t)\}=\left\{\bar{Y}(t)\right\}=[\Phi]\{Z(t)\}=\sum_{j=1}^{2n}\{\bar{f}\}_j^T z_j(t)$$

where an upper dash indicates the half lower part of a given vector.

This displacement response vector was used by Maldonado and Singh (1992) to obtain an estimate of the maximum value, M_R, of any response quantity, R, linearly related to these relative displacements. For this purpose they used ground response spectrum values to model the seismic input and used the MMD approach. Their expression for the square value of such maximum is:

$$M_R^2=C_s^2G^2+4C_s\sum_{j=1}^{r}\left[\frac{z_jS_{paj}^2}{w_j^2}+\left(d_jb_jw_j-z_j\right)S_{pvj}^2\right]+4\sum_{j=1}^{r}\left[\frac{z_j^2S_{paj}^2}{w_j^4}+d_j^2S_{pvj}^2\right]+$$

$$8\sum_{j=1}^{r-1}\sum_{k=j+1}^{r}\left[\frac{T_{jk}^I}{w_j^4}\left(S_{paj}^2-S_{pak}^2\right)+T_{jk}^{II}\left(S_{pvj}^2-S_{pvk}^2\right)+\frac{z_jz_k}{w_j^4}S_{pak}^2+d_jd_kS_{pvk}^2\right]$$

where:

w_j is the j^{th} frequency of the structure.

G is the maximum ground acceleration.

S_{paj} is the pseudo-acceleration spectrum value.

S_{pvj} is the pseudo-velocity spectrum value.

C_s is the response of the truncated modes.

z_j is the real part of the complex quantity $P_j^c(w)$.

T_{jk}^{I} and T_{jk}^{II} are partial fraction coefficients .

2.2. Rate of change of the response.

The last expression was used as a basis for developing the proposed formulation that estimates the rate of change of the maximum value M_R of a given response quantity R. This rate is with respect to the damping coefficient c_i of an added damper. It depends on several other variables and their respective rates. That is, on the rates of eigenvalues, eigenvectors, frequencies, damping ratios and on the rates of all other variables involved in the expression for the response by the MMD approach.

After some algebraic manipulations, these rates of change are:

Rates of change for eigenvalues:

$$\frac{\partial l_j}{\partial c_i} = -\left\{ \{f_j\}_L^T \left(l_j^2 \left[\frac{\partial M}{\partial c_i}\right] - l_j \left[\frac{\partial C}{\partial c_i}\right] + \left[\frac{\partial K}{\partial c_i}\right] \right) \right\} \{f_j\}_L$$

Rates of change for eigenvectors:

$\dfrac{\partial \{f_j\}}{\partial c_i} = \sum_{l=1}^{2m} a_{jl}\{f_l\}$ where the value for a_{jl} when j=l is:

$$a_{jl} = -\frac{1}{2}\{f_j\}_L^T \left(-2 l_j \left[\frac{\partial M}{\partial c_i}\right] + \left[\frac{\partial C}{\partial c_i}\right] \right) \{f_j\}_L$$

and for j different from l a_{jl} is:

$$a_{jl} = \left(\frac{1}{l_l - l_j}\right) \left\{ \{f_j\}_L^T \left(l_j^2 \left[\frac{\partial M}{\partial c_i}\right] - l_j \left[\frac{\partial C}{\partial c_i}\right] + \left[\frac{\partial K}{\partial c_i}\right] \right) \right\} \{f_j\}_L$$

Rates of change for natural frequencies:

$$\frac{\partial w_j}{\partial c_i} = \frac{\left(z_R \dfrac{\partial z_R}{\partial c_i} + z_I \dfrac{\partial z_I}{\partial c_i} \right)}{w_j}$$

Rates of change for damping ratios:

$$\frac{\partial b_j}{\partial c_i} = \frac{w_j \dfrac{\partial z_R}{\partial c_i} - z_R \dfrac{\partial w_j}{\partial c_i}}{w_j^2}$$

The rate of change of the squared maximum value of a given response quantity is:

$$\frac{\partial M_R^2}{\partial c_i} = 2 M_R \frac{\partial M_R}{\partial c_i}$$

and the expression corresponding to $\frac{\partial M_R}{\partial c_i}$ is:

$$\begin{aligned}
\frac{\partial M_R^2}{\partial c_i} &= 2 C_s C_s' G^2 \\
&+ 4C_s \sum_{j=1}^{r} \left[\begin{array}{l} \dfrac{w_j^2\left(2z_j S_{paj} S_{paj}' + S_{paj}^2 z_j'\right) - 2z_j S_{paj}^2 w_j w_j'}{w_j^4} + \left(d_j' b_j w_j + d_j b_j' w_j + d_j b_j w_j' - z_j'\right) S_{pvj}^2 \\ +2\left(d_j b_j w_j - z_j\right) S_{pvj} S_{pvj}' \end{array} \right] \\
&+ 4C_s' \sum_{j=1}^{r} \left[\frac{z_j S_{paj}^2}{w_j^2} + \left(d_j b_j w_j - z_j\right) S_{pvj}^2 \right] + 4\sum_{j=1}^{r} \left[\begin{array}{l} \dfrac{w_j^4\left(2z_j^2 S_{paj} S_{paj}' + 2S_{paj}^2 z_j z_j'\right) - 4z_j^2 S_{paj}^2 w_j^3 w_j'}{w_j^8} \\ +2d_j^2 S_{pvj} S_{pvj}' + 2S_{pvj}^2 d_j d_j' \end{array} \right] \\
&+ 8\sum_{j=1}^{r-1} \sum_{k=j+1}^{r} \left[\begin{array}{l} \dfrac{w_j^4 T_{jk}^{I'} - 4T_{jk}^{I} w_j^3 w_j'}{w_j^4}\left(S_{paj}^2 - S_{pak}^2\right) + \dfrac{T_{jk}^{I}}{w_j^4}\left(2S_{paj} S_{paj}' - 2S_{pak} S_{pak}'\right) \\ +T_{jk}^{II}\left(2S_{pvj} S_{pvj}' - 2S_{pvk} S_{pvk}'\right) + T_{jk}^{II'}\left(S_{pvj}^2 - S_{pvk}^2\right) + \dfrac{w_j^4\left(z_j z_k' + z_j' z_k\right) - 4z_j z_k w_j^3 w_j'}{w_j^8} S_{pak}^2 \\ +2S_{pvk} S_{pvk}' \dfrac{z_j z_k}{w_j^4} + d_j' d_k S_{pvk}^2 + d_j d_k' S_{pvk}^2 + 2d_j d_k S_{pvk} S_{pvk}' \end{array} \right]
\end{aligned}$$

where a comma over a variable indicates the rate of change of such variable with respect to the damping coefficient, c_i, of the added damper.

The seismic input forces are defined in terms of well-known ground response spectral values. In particular, the pseudo-acceleration and pseudo-velocity spectra need to be used. Example of the pseudo acceleration spectra are given in Figure 3.

3. IMPLEMENTATION

Currently, the above expressions are being incorporated into a special computer program written in Matlab software. The program considers a three-dimensional frame structure with added dampers. It performs a double eigenanalysis. The first one is a classic analysis needed to determine the classical damping matrix (proportional). The second eigenanalysis is performed in the complex domain and is required to decouple the modified equations of motion after the incorporation of the added dampers. Therefore, the program not only allows the use of extra dampers but also the use of proportional damping. This program is still in the developmental stage. It will be properly debugged and verified. It is expected that the proposed formulation will provide good estimates of the response quantities and their rates of change with respect to the damping coefficients of the added dampers. This rate is crucial to perform a sensitivity analysis of a given response to the addition of dampers. It is expected that such sensitivity analysis will constitute a valuable tool for designing structures with the beneficial effects of added dampers.

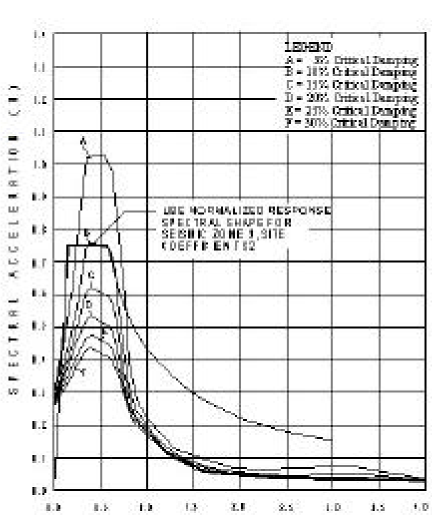

Figure 3. Ground response spectra.

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