Effect of Eccentricity on Electric Field Profiles in Ellipsoids

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Abstract

We study the effect of shape on the electric field profiles in ellipsoids, by means of the T-Matrix method. We considered two non-concentric ellipsoidal objects; one completely embedded inside the other. By varying the semi-axes of the smaller ellipsoid, we mimic various tumor eccentricities (maintaining a constant volume) in the brain. The results are of interest as a diagnostics tools for clinical techniques such as magnetic resonant imaging (MRI), since it is well known that the morphology of the tumor determines its degree of malignance [1].

1. Introduction

Since its inception by P.C. Waterman as a technique to calculate electromagnetic scattered fields, T-Matrix [2,3], also called "Extended Boundary Condition" method, has been applied in different kinds of electromagnetic problems [4]. Later on, Peterson and Ström contributed to the method by providing the capability to handle multilayers and an arbitrary number of scatterers [5,6]. The algorithm that we used in this paper, to calculate the electromagnetic fields, is based on those results.

T-Matrix expresses the incident, scattered, and internal fields as corresponding multipolar vector expansions. Each order of expansion is given in terms of spherical Bessel, and Hankel functions.

The expansion coefficients and the boundary conditions are used to construct the matrices that relate known field with unknowns ones. The elements of those matrices are surface integrals on the various boundaries, of expressions containing the multipolar functions.

The convergence of this method is strongly dependent on the system geometry, frequency, and electromagnetic properties of the scatterers [7]. As the values of the parameters vary, T-Matrix may include higher multipolar orders. For example, we find that as the frequency increases, so does the number of multipoles necessary to maintain a constant accuracy. On the other hand, the dimensions of the matrices increase as a quadratic function of the order [7,8]. Thus, it would seem as if one needed a supercomputer to pursue our project. However, it turned out that we obtained results of excellent accuracy on a Pentium I, 90 MHz, 64 Mbytes RAM in 10

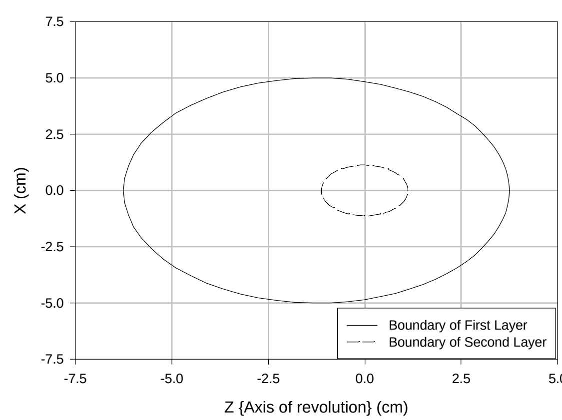

Figure I

hours for a multipolar order of 10. The time was significantly reduced to 1 hour when a

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Pentium II, 450MHz, 500 Mbytes, was used [7].

2. Description of the System

The system configuration consists of two ellipsoidal objects; one completely embedded inside the other. Figure I provides a graphical illustration of the system. The larger ellipsoid represents the gray-white matter of the human brain. The smaller, embedded object models a tumor growth. The larger object is kept fixed. The smaller object is considered for various eccentricities since it is well know that the morphology of the tumor is directly related to its malignancy[1]. In order to isolate the effects due to shape, as opposed to just size, we kept the volume of the tumor at a constant value. For each shape (eccentricity) we calculate the electromagnetic field profiles, and look for changes. The electromagnetic field profiles affect the MRI reconstruction algorithm. Therefore, the results presented here are of use as a diagnostic tool in tumor evaluation.

The relationship between the equatorial and axial semi-axes, maintaining a constant volume of the inner ellipsoid is:

$$a = \sqrt{3}\sqrt{\frac{b^2V\left(16b^3p + \sqrt{3V(32b^3p + 3V)}\right)}{64b^6p^2 + 48b^3Vp + 18V^2}}$$

where a is the vertical semi-axis, b is the horizontal semi-axis, and V is the volume (considered here to be a constant) of the smaller ellipsoid.

For our configuration we consider the volume of the small ellipsoids to be equal to that of a sphere with radius equal to 1.5 cm, that is $V=14.14\ cm^3$.

3. Validation and Numerical Results

The validation of the code was done in the electrostatic limit for the case of a dielectric sphere in the present of an incident plane wave. The standard solution is known to be [9].

$$E^i = \frac{3E^o}{e + 2}$$

where E^i is the electric field inside the sphere, E^0 is the *incident* electric field, and ε is the dielectric constant of the sphere. For the test we used a permittivity 60, obtaining an agreement between the theory and computer algorithm of 10^{-5}%.

The properties of the outer and inner layers are $\varepsilon_1 = 46.25$, $\sigma_1 = 1.85$ S/m, $\varepsilon_2 = 60$, $\sigma_2 = 1$ S/m [10-13]. With those properties we studied the effect of shape on electric fields for various frequencies and eccentricities. Here we only show results for frequencies relevant to magnetic resonant imaging. Typical modern clinical systems run at 64 MHz (1.5 T for water). Experimental runs are done today at frequencies as high as 700 MHz. Table I contains the legends of the plots that identifies the eccentricity of each data.

Table I

Legend	Eccentricity
————————	1
………………………	0.708

In the figures we observe clearly the difference between the signals across the tissue for the two eccentricities considered here for the inner ellipsoid. We can establish that the discontinuities vary with the morphology of the inner tissue (Figures III, V). Also we obtained changes of the signal with frequency (Figures II, IV).

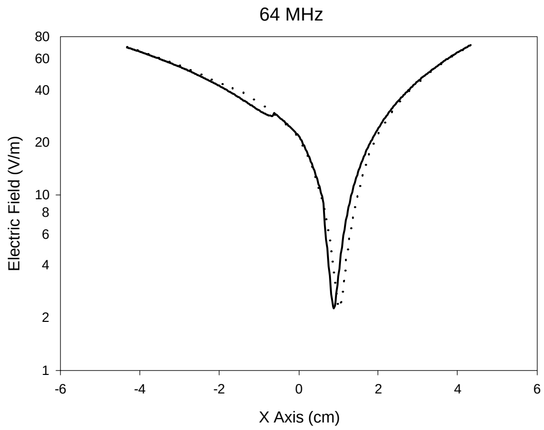

Figure II. Magnitude of electric field along the X axis at 64MHz.

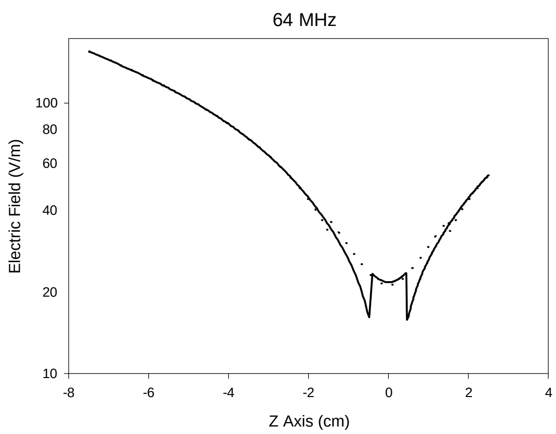

Figure III. Magnitude of electric field along the Z axis at 64MHz.

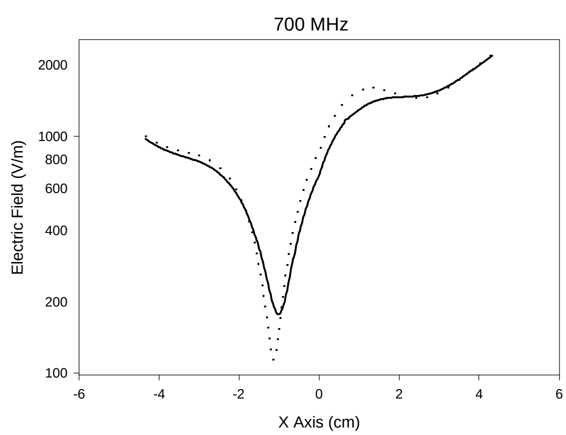

Figure IV. Magnitude of electric field along the X axis at 700MHz.

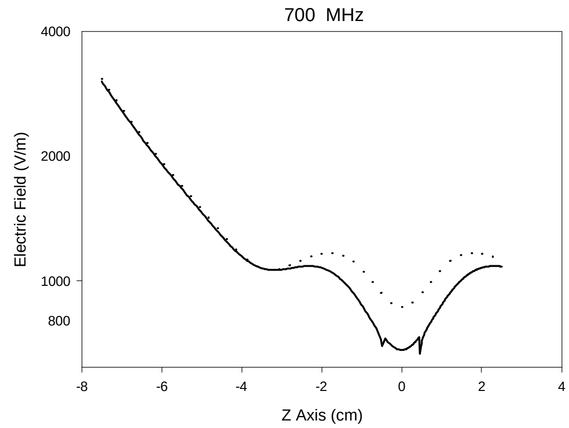

Figure V. Magnitude of electric field along the Z axis at 700MHz.

4. Conclusions

We calculated electric field profiles for various frequencies and tumor shapes. The motivation is that these results can be used in the reconstruction algorithms of MRI and related clinical techniques. By means of the electromagnetic field profiles one can determine the place and the morphology of the tumor embedded in healthy tissue.

5. Future Work

We are setting up more realistic configurations for tumor morphology. In particular we are considering the surface roughening.

Acknowledgment

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