Ising Model for Image Analysis

Jose C. Ramos Barreto
Fernmarie Rodriguez
Advisor: Dr. Robert Acar

Laboratory for Applied Remote Sensing and Image Processing
Electrical and Computer Engineering Department
Physics Department
University of Puerto Rico, Mayagüez Campus
Mayagüez, Puerto Rico 00681-5000

jcramos@mail.com
fernmarie@hotmail.com

ABSTRACT

One goal of low-level vision is to segment the domain into distinct surface patches belonging to different objects in the scene. A probabilistic approach is to predict how likely a given segmentation is, on the basis of low-level information. The likeliest segmentation is the one which minimizes the total "energy", sum of a prior energy and a data-dependent term.

The Ising model was introduced to model phase transitions in statistical physics; in vision, it is used mostly to model bichromatic scenes. We describe how to implement this model using genetic algorithms for one-dimensional images.

1. INTRODUCTION

In order to get a low-level vision we may segment a domain **D** of an image **I** into parts, on which distinct surface patches, belonging to distinct objects in the scene, are visible. We come up with probability measures, in the domain, of how likely a given segmentation is on the basis of all available low-level information, and what is the most probable segmentation. We seek the segmentation with minimum energy. One may set the "energy" of the segmentation to be: E (I,S)

$$E(w) = Ed(I,w) + Ep(w)$$

where we explain the meaning of the symbol S below.

If we regard the possible segmentations as the samples of our experiment, our energy incorporates two sources:

- A *prior term (Ep)* of possible scene segmentations, and
- A *data term (Ed)* incorporating a given data image.

where the goal is to minimize:

E (w).

2. THE ISING MODEL

Binary images may be regarded as white blobs against dark background (or vice versa). If one regards an image as a function taking values in the range [0,1] (intensity of each pixel), binary images correspond to characteristic functions of sets S, where the intensity is 1 (white blobs).

Given a gray scale intensity image I

$w = S$.

$Ep = n \mid \partial S \mid$.

$Ed = \iint (I - c_S)^2 \, d\vec{x}$.

Where:

$\mid \partial S \mid$ - is the length of the boundary of S.

ν – scalar coefficient.

The blobs are supposed to be characterized by their specific brightness in contrast to the background:

I is modelled by Xs + n where n is white noise.

What the Ising model seeks to correct is precisely the substantial amount of noise present in the images.

3. GENETIC ALGORITHMS

The genetic algorithm is our optimization method or tool, it is not a particular algorithm, but rather, a set or group of methods which seek to minimize a function defined on a domain by:

- Discretizing a domain so that each vector of fixed length N (chromosome) codes value of the position variable.
- Arbitrarily selecting an initial population of chromosomes by sampling from the domain and using an equal distribution.
- Allowing this initial population to evolve over a specific number of generations. One of the best-fit members of the last generation is then taken as our solution. The evolution rules, may vary as follows:

  a. The selection: based on fitness, they arbitrarily select couples to reproduce.

  b. Each couple will generate one, or alternatively two offsprings by randomly creating a "crossover" rule.

  c. A desirable step at this point is to randomly mutate some of the offsprings.

  d. The offsprings will now make a new population, which is of the same size as the previous population.

Note that direct methods for minimizing the energy fail miserably because it is a highly non-convex functional.

4. CURRENT PROGRESS

We have implemented the minimization of the one dimensional Ising model of an image using the genetic algorithm method. We have accomplished our current project using MATLAB. Our control parameters include: size of population, number of generations, probability of crossover, probability of mutation and the value of ν, coefficient of the prior term. For the purpose of this experiment we varied: size of population, number of generations and ν. The values of the fixed parameters were: probability of mutation = 1/population and probability of cross over = .7.

In figure 1 we have plotted our Image in one dimension.

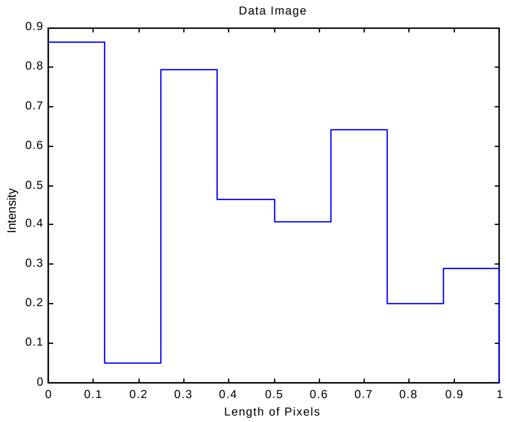


Figure 1. Data Image

Figure 2 illustrates that the performance of the energy depends strongly on ν.

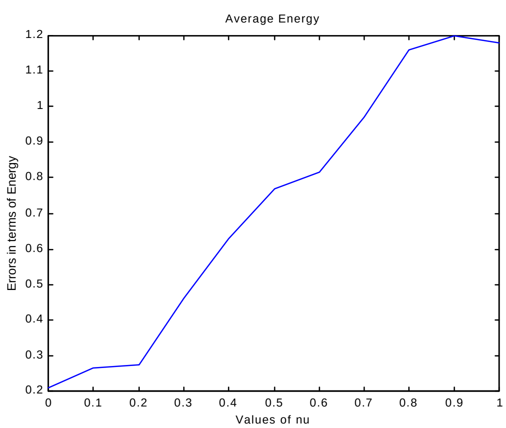
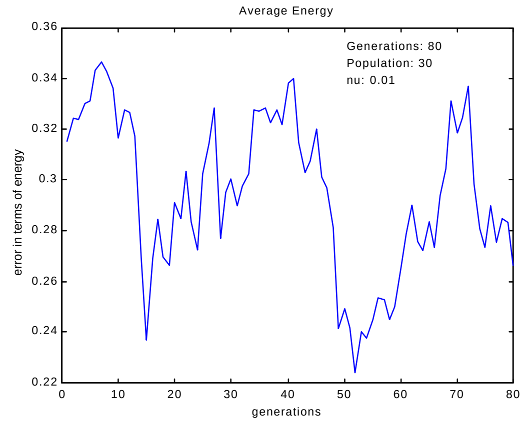
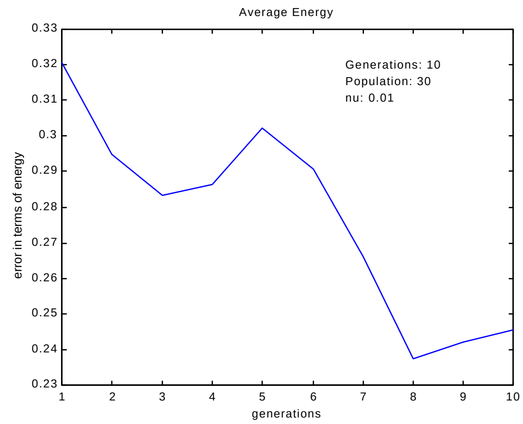


Figure 2. Energy versus ν plot.

In addition, in order to achieve a better approximation, the number of generations should be 20 or lower (see figure 3). The population should range around 30 or more (see figure 4).

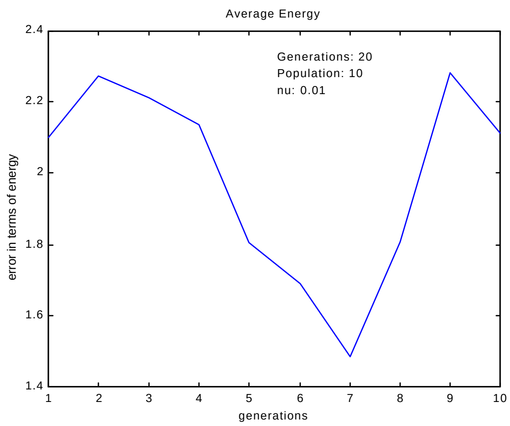


a.

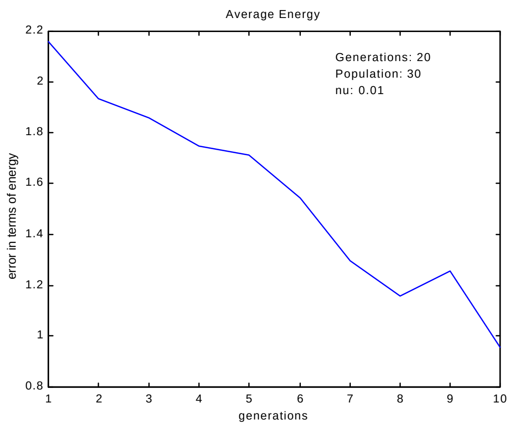


b.

Figure 3. The population and the value for ν, were held fixed. a. The generations were set to 80. b. The generations were set to 10.



a.



b.

Figure 4. The generations and the value for ν, were held fixed. a. The population was set to 10. b. The population was set to 30.

FUTURE WORK

Although we do not expect the genetic algorithm to outperform the iterative method, we are certain that it approximates the solutions with accuracy. Our next step in the one dimensional case is to find criteria to select the value of ν. In addition we will implement the Ising Model and Genetic Algorithm to minimize the energy functional for the 2-dimensional images.

ACKNOWLEDGEMENT

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