Image Evolution Using the Wave Operator

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Abstract

PDE methods in image processing operate by evolving the image according to some differential operator. Anisotropic diffusion, meaning any nonlinear analogue of diffusion by the heat operator, is used both for the purpose of segmentation and edge preservation; a multi-scale representation of the image is obtained in function of time. The purpose of this work is to see how some properties of the image may be captured, strengthened or changed, by running the wave operator coupled with the usual anisotropic diffusion.

1. INTRODUCTION

There is a need to form from original images blurred counter parts in order to smooth out any discrepancies, but in doing so we wish not to alter the edges of the image, edges being natural boundaries of objects of the scene. Smoothing is usually done using diffusion. From the heat equation an isotropic diffusion could be modeled to fit our needs but such diffusion has no inherent respect for edges: it ends up smoothing out the entire image which is an undesirable result. On the other hand, anisotropic diffusion as given by the Perona-Malik equation permits us to maintain our edges. Using this in conjunction with the wave operator as proposed by S. Kichenassamy, we want to investigate the way in which certain geometric patterns in the image are modified while still allowing diffusion to smooth out the noise.

2. IMPLEMENTATION

At this stage of the investigation we began with one dimensional images in order to gain experience with the simplest case, keeping in mind that the properties of the wave operator present some differences in higher dimensions. First we apply the wave operator to our original image; we allow it to propagate maintaining adiabatic boundary conditions (these conditions are consistent with the boundary conditions of the diffusion step). Once propagated for time T, an anisotropic diffusion takes the output and smoothes out the image. Then we allow the wave operator to run backward for the same time T, yielding a final image(output). We then compare the input and output in the presence or absence of the diffusion middle step.

Case 1.a

Our first experiment consists in running the wave operator forwards, skipping the diffusion step, then running the wave operator backwards a function without oscillations.

In figure 1.1 we can see the original input which represents a unit step function.

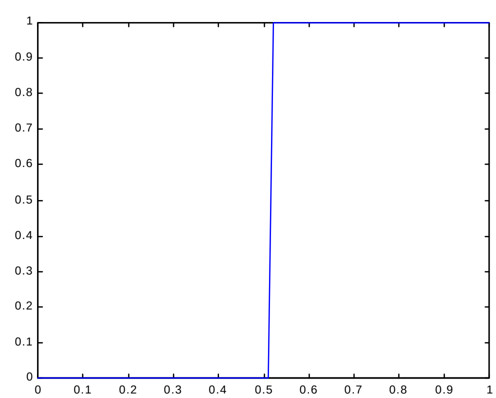

Figure 1.1 Original Input

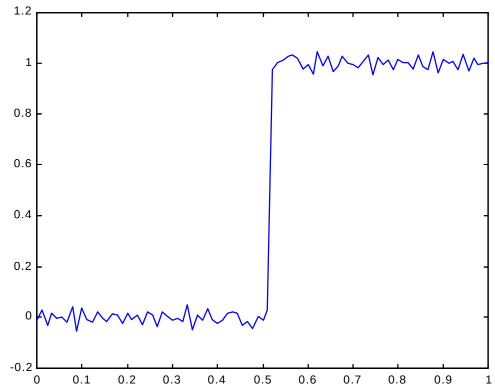

Figure 1.2 Output of Case 1.a

In figure 1.2 we can see a plotted representation of what is considered as a result from the program what is important to notice is that this run is produced with out any anisotropic diffusion, even though we can perceive waves in the result this is an inherit consequence of using a finite difference method to estimate the function.

Figure 1.3 represents the errors associated in reaching the final output.

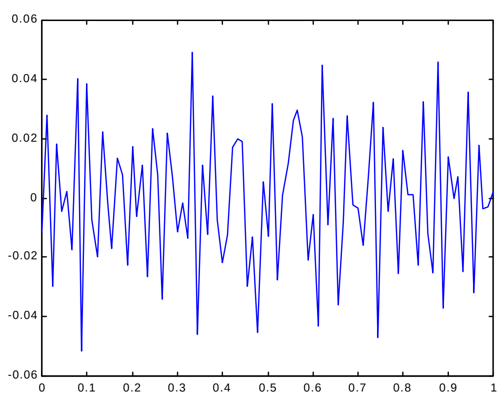

Figure 1.3 Errors in Reaching Final Output of Case 1.a

Case 1.b

The second part of the experiment consist of running the wave operator with anisotropic diffusion as a middle step to serve as input for the backwave function with equal amount of parameters as Case 1.a. The input is same as figure 1.1.

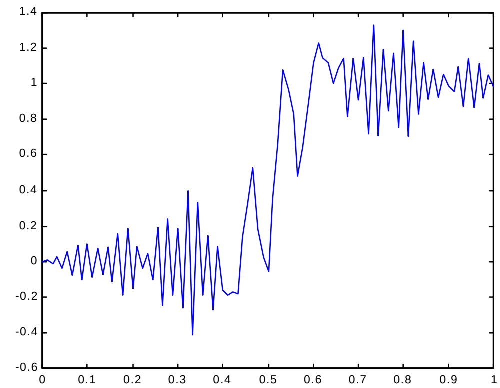

Figure 1.4 Output of Case 1.b

In figure 1.4 we can see a plotted representation of our output with an anisotropic diffusion middle step.,

Case 2.a

Our Second experiment consists in running the wave operator forwards, skipping the diffusion step, then running the wave operator backwards on a function with oscillations.

In figure 2.1 we can see the original input which represents a unit step function with oscillations.

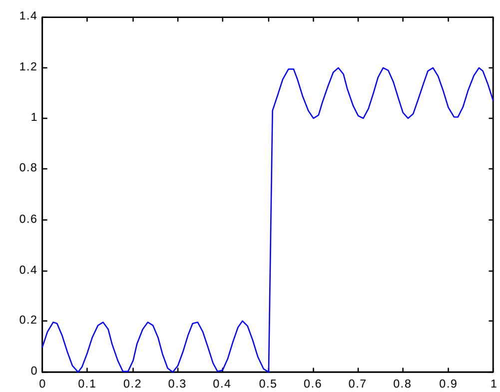

Figure 2.1 Original Input of Case 2.a with Oscillations

In figure 2.2 we can see a plotted representation of the output it is important to notice that this run is produced without any anisotropic diffusion.

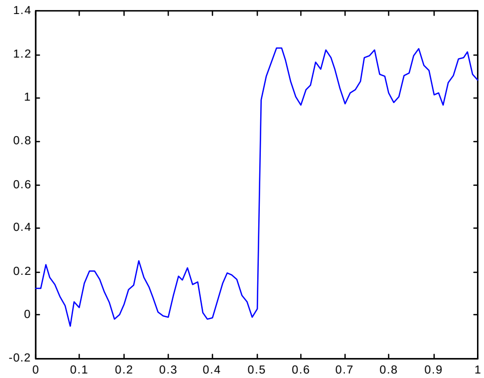

Figure 2.2 Output of Case 2.a

Figure 2.3 represents the errors associated with the final output. Although very jagged remember to pay attention to the scale where errors are not more than ±.06

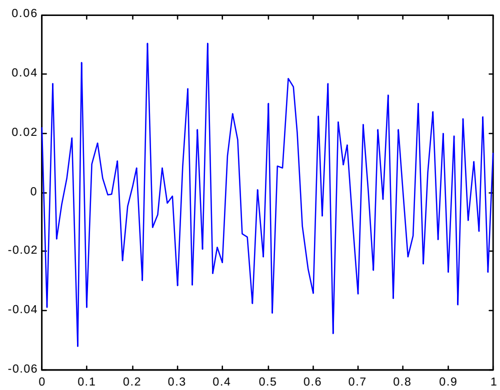

Figure 2.3 Errors in Reaching Final Output of Case 2.a

Case 2.b

The second part of the experiment consist of running the wave operator with anisotropic diffusion as a middle step to serve as input for the backwave function with equal amount of parameters as Case 2.a. The input is same as figure 2.1.

In figure 2.4 we can see a plotted representation of our output with an anisotropic diffusion middle step.

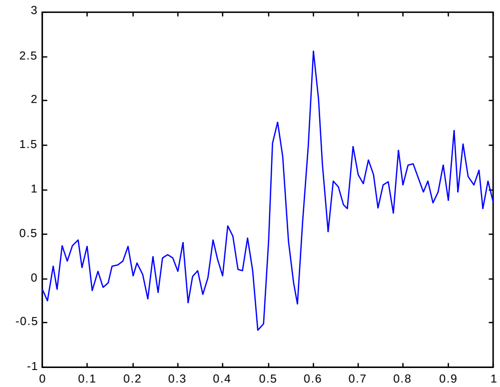

Figure 2.4 Output of Case 2.b

3. CONCLUSIONS AND FUTURE WORK

The results obtained are somewhat unexpected; since one would expect that the middle step would help remove the noise (fast oscillations), instead it seems to increase it. The observed oscillations can be ascribed to two factors; the effect of the finite difference scheme, and the effect of the wave operator itself. We mostly suspect the former. This being the case, we next plan to implement the wave operator using the method of characteristics (d'Alembert solution) instead of finite differences, and compare the solutions. We will then either continue to the two-dimensional case, or test other wave like evolution operators in the one-dimensional case.

ACKNOWLEDGEMENT

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